Durability Management of a Complex System

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Abstract: I examine the complex system durability control method with the usage of the modelled change of the failure probability, with the passing of time and the degrading of the system. I propose an approach to the problem of the accounting of the relations between the complex system blocks. I describe a modelling algorithm that combines the logical linguistic and logical probabilistic prognosis of the processes of the parameter values' change with the passing of time. The modelling allows getting a time reserve to run the needed maintenance operations, increasing the reliability of the system.

Keywords: complex logical function, complex system maintenance, logical linguistic modelling, prognosis logical probabilistic modelling.

I. INTRODUCTION

The main principles that I use for the complex systems durability control are adaptation, dynamic natural selection (or hot reservation), stress, compensation and borrowing, stupor or enabling of the emergency mode [4]. For the inner system state deviations due to various failures, to achieve the needed durability I traditionally use the hot reservation principle, which is similar to the natural dynamic selection in the living organisms. A signal to enable the mechanism of the dynamic natural selection, that is the switching the channels and the blocks to the spare ones, is the observed overlaps of the blocks' inner state, that can be measured by its expected value of the block's parameters, or by its failure probability [3].

The problem of the system's durability or its reliable functioning provisioning, when the system's inner state deviation exceeds its allowed thresholds, is stated quite a long time ago and is mostly examined [6]. However, while estimating the change with the passing of time of a complex logical function, which describes the system's failure probability with the accounting of the relations between the blocks (excluding only the simplest schemes), there appear certain complexities and ambiguities [2]. The problem of the accounting of the parameters of the system's blocks' influence on the parameters of the blocks they are related to, while calculating the failure probability with the passing of time of the complex system still doesn't have a practically acceptable solution [5], since the analytical account of that issue in a complex system invariably leads to very complex computations. Let us examine one of the possible approaches to the problem of the relations' accounting between the blocks of a complex system.

II. A SIMPLIFIED ACCOUNTING OF THE RELATIONS BETWEEN THE BLOCKS OF A COMPLEX SYSTEM

It is clear that with the passing of usage time T of a complex system the probabilities of correct functioning of its blocks $P_{ic}(T)$ are decreasing by an exponential law [1]:

$P_{ic0}(t_i) = \exp(-{\sim}_{i0} t_i)$ (1)

Where t_i is the usage time of the ith block of the system, a_{i0} is the decrease coefficient, which I find out of the equation (1), since the mean time between failures t_{i0} and the initial correct functioning probability $P_{ic}(T)$ are usually given for the system blocks.

That decrease of the probabilities may be described by the following change of their parameter values' expected values [1]:

$$P_{i0}(T) = 1 - \Phi((b_i - m_i)/\sigma_i) + \Phi((-b_i - m_i)/\sigma_i) = \\ = 1 - P_{ic0}(t_i) \quad (2)$$

Where: b_i is the maximum allowed value of the *i*th parameter.
m_i is the expected value of the *i*th parameter.
σ_i is the root mean square of the *i*th parameter.
$\Phi(x)$ is the probability integral that cannot be expressed through elementary functions, but there are tables of its calculated values [1], or its approximate value can be found as a sum of a decreasing row.
Since the initial values for $P_{iбо}$, b_i and m_{i0} are usually known for each block, the root mean square value σ_i for each block may be found from the following equations:

$$P_i(-\infty < x_i < -b_i) = \Phi_i((b_i - m_{i0})/\sigma_i) - \Phi_i(-\infty) = \\ = 1 - P_{ic0}(t_i) \quad (3),$$

$$\Phi(-\infty) = 0 \quad (4),$$

It is also clear that the approaching of the expected values of the *i*th block's parameters to the dangerous (critical) threshold c_i and, furthermore, to the maximal allowed threshold b_i also affect the parameters of the related blocks. For instance, the change in the power supply block's output voltage also affects the amplification coefficient of the related amplification block. However, the problem of the estimation of the change with the passing of time of a complex logical function, which describes the system's failure probability with the accounting of the relations between the blocks still doesn't have a practically acceptable solution [6], since the analytical accounting of that fact in a complex system leads to very complex computations. Thus, I propose the following simplified approach to that problem:
When the expected value $m_i(t_i)$ of the block's parameters in some time moment t_{ik} fall into a dangerous zone $c_i \le |m_i| < b_i$, I set the coefficients $w(i) = 2$, $u(i) = 3$ for this block. Here,
$w(i)$ is a state characteristic of the *i*th block ($w(i) = 3$ is broken, $w(i) = 2$ is dangerous, $w(i) = 1$ is normal).
$u(i)$ is the characteristic of the *i*th block proximity to the nearest broken or dangerous block ($u(i) = 0$ is far, $u(i) = 1$ is connected via a single block, $u(i) = 2$ is directly connected, $u(i) = 3$ is self).
After, I perform an expected value shift:

$$m_i^* = m_i + \sigma_i w(i) u(i) m_i \mu(m_i) \quad (5)$$

Where $\mu(m_i)$ is the current expected value membership function of a certain interval, which I calculate as following (Fig. 1):

If $-\infty < m_i < b_i + m_{i0}$, then $\mu(m_i) = 1$

If $-b_i + m_{i0} \le m_i \le -c_i + m_{i0}$, then $\mu(m_i) = \max\left\{ \begin{matrix} (m_i - m_{i0} + c_i)/(c_i - b_i); \\ (m_i - m_{i0} + b_i)/(b_i - c_i) \end{matrix} \right\}$

If $-c_i + m_{i0} \le m_i \le m_{i0}$, then $\mu(m_i) = \max\left\{ \begin{matrix} (-m_i + m_{i0})/c_i; \\ (m_i - m_{i0} + c_i)/c_i \end{matrix} \right\}$

If $m_{i0} \le m_i < c_i + m_{i0}$, then $\mu(m_i) = \max\left\{ \begin{matrix} (-m_i + m_{i0} + c_i)/c_i; \\ (m_i - m_{i0})/c_i \end{matrix} \right\}$

If $c_i + m_{i0} \le m_i \le b_i + m_{i0}$, then $\mu(m_i) = \max\left\{ \begin{matrix} (m_i - m_{i0} - b_i)/(c_i - b_i); \\ (m_i - m_{i0} - c_i)/(b_i - c_i) \end{matrix} \right\}$

If $b_i + m_{i0} \le m_i < \infty$, then $\mu(m_i) = 1$

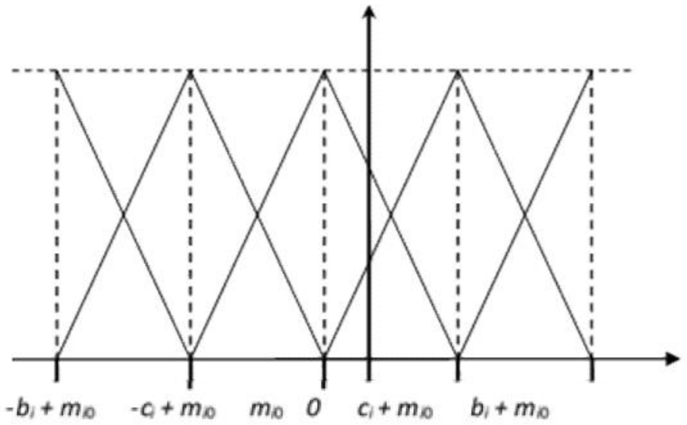

Figure 1. Fuzzification

Then I set the block numbers j of the blocks that are directly related to the "dangerous" block. For them, I set the values of the relation coefficients $u(j)=1$, $u(j)=2$ and perform the expected value shift:

$$m_j^* = m_j + \pm_j w(j)u(j)m_j «(m_j) \quad (6)$$

Where I calculate $\mu(m_j)$ by the same rules 1) – 6).

After, I define the block numbers q, that are connected to the block i via a single block. For them, I set $w(q)=1$, $u(q)=1$ and perform the expected value shift:

$$m_q^* = m_q + \pm_q w(q)u(q)m_q «(m_q) \quad (7)$$

If now, after recounting of the expected values, I find that some block has its absolute value over the allowed threshold ($|m_i| > b_i$), such block is accounted to be broken, its failure probability is set to one $(P_{fi}=1)$, and the whole system's failure probability is set to one $P_f=1$. Otherwise, I need to calculate the new values of failure probability of all blocks from the new expected values, according to the formula (2), and then to calculate the failure probability of the whole system, by using, for instance, a polynomial formula [6]:

$$\begin{aligned} P_f &= (-1)^0 \sum_i (P_{if}(T)) + (-1)^1 \sum_{ij} (P_{if}(T)P_{jf}(T)) + \\ &+ (-1)^2 \sum_{ijk} (P_{if}(T)P_{jf}(T)P_{kf}(T)) + \ldots + \prod_i (P_{if}(T)) \end{aligned} \quad (8)$$

Thus in the proposed solution to the relations' accounting problem, when a dangerous situation arouses, I instantly change the expected parameter values of the given block and its related blocks, which allows in a first approximation to account the mutual impacts of the blocks' parameters to the change of the failure probabilities with the passing of time.

III. The modelling of changes with the passing of time of the complex system failure probability with the reservation of the blocks

While modelling the change with the passing of time of the complex system failure probability with the reservation of the blocks I assume that the system contains N_b basic and N_r reserve blocks. I must determine the change with the passing of time t of the failure probability $P_s\{y = 1\}$ of the system, if I know:

The structure of the system, by which I can draw the block relations table.

The mean time between failures for each ith block of the system t_{i0}.

The initial probability of the correct functioning of each ith block of the system $P_i(t_{i0})$.

The maximal (critical) deviation threshold of the ith block parameters b_i. When this threshold is surpassed, I assume that the block is broken ($z_i = 1$ – ith block failure).

The dangerous deviation of the parameters of the *i*th block c_i. When surpassed, this leads to the increase of the parameters' deviation of related blocks.
The deviation threshold d_i, surpassing of which leads to the substitution of the block with the reserve one.
In a system with reservation the system failure means that:

$$y = (z_{1b} \wedge z_{1r}) \vee (z_{2b} \wedge z_{2r}) \vee (z_{3b} \wedge z_{3r}) \vee ... \vee (z_{Nb} \wedge z_{Nr}) \quad (9)$$

Where z_{ib} is the failure of *i*th basic block and z_{ir} is the failure of the *i*th reserve block of the system.
Thus to compute the system's failure probability that characterizes its reliability, in a time moment T I must first compute the probabilities of the conjunctive elements in the equation (9):

$$P_i\{z_{ib} \wedge z_{ir} = 1\} = P_{ib}\{z_{ib} = 1\} * P_{ir}\{z_{ir} = 1\} = ... = = P_{ib}(T_{ib}) * P_{ir}(T_{ir}) = P_i(T) \quad (10)$$

Where T_{ib}, T_{ir} are the corresponding working time of the *i*th basic and reserve blocks.
Furthermore:
Since I are interested in the situations when the blocks' parameters are nearing the dangerous values, when $c_i < |m_i| < b_i$, where c_i is the dangerous parameter threshold and b_i is the critical threshold of the parameter, or the critical values, when $|m_i| \geq b_i$, I may ignore the value $\Phi((-b_i - m_i)/\pm_i)$ in the equation $P_0(T) = 1 - \Phi((b-m)/\pm)) + \Phi((-b-m)/\pm_i$, since $\Phi((-b_i - m_i)/\pm_i) < \langle \Phi((b_i - m_i)/\pm_i)$. Thus I may assume that $P_{i0}(T_{i0}) = \Phi((b_i - m_{i0}(T_{i0})/\pm_{i0})$ and $P_{ip}(T_{ip}) = \Phi((b_i - m_{ip}(T_{ip})/\pm_{ip})$. If the *i*th reserve block is not connected, $T_{ir} = 0$ and $P_{ir}(T_{ir}) = 0$.
If I don't have the *i*th reserve block, then $P_{ir}(T_{ir}) = 1$
If some *i*th basic block doesn't have a reserve one and for that block $P_{i0}(T_{ib}) = 1$ block failure, then $P_c\{y = 1\} = 1$
If I don't have any *i*th block, for which $P_{ib}(T_{i0}) = 1$ and $P_{ir}(T_{ir}) = 1$ then:

$$P_c\{y=1\} = P_c(T) = (-1)^0 \sum_i (P_i(T)) + (-1)^1 \sum_{ij} (P_i(T) P_j(T)) + + (-1)^2 \sum_{ijk} (P_i(T) P_j(T) P_k(T)) + \ldots + \prod_i (P_i(T)) \quad (11),$$

If for some *i*th block in some *ij*th moment of time *Tij* the parameter deviations exceed the threshold d_i, then, providing that I have a reserve block, I change it by that block, for him I set the value $P_{i0}(T_{ib}) = 1$ and for the reserve block I set $T_{ir} = T - T_{ij}$. Then, $P_i(T) = P_{ir}(T_{ir})$, и $P_c\{y = 1\}$ I also compute by the formula (11).
If after replacing the basic block to the reserve one I have $P_{ir}(T_{ir}) = 1$, then $P_c\{y = 1\} = 1$.
The correct functioning probability of each *i*th block decreases with the passing of its working time t_i according to the exponential law $P_{ib}(T_{ib}) = \exp(-a_{i0}\, T_{ib})$, (10), and the probability or correct work of each *i*th reserve block $P_{ir}(T_{ir})$ remain constant until the time moment of their enabling T_{ij}, and then decrease with the time passing $T_{ir} = T - T_{ij}$ also by the exponential law of the type (1).

IV. The algorithm of the modelling of the failure probability with the passing of time of the complex system with the reservation of the blocks

Taking into account the facts described above, I compose the following algorithm:

1) *Initializing*

N – the quantity of blocks of the system
$\Phi(x)$ – the normal distribution function, given as a linearly interpolated table values [1]
V – system time; $V := 0$
T – final modelling time
Δ – modelling time step
C – square binary matrix of rank N, which describes the pointed graph of the system's blocks relations.
$m_i(V)$ – expected value of the *i*th block condition before accounting the relations in a time moment V

$Mi(V)$ – addition to the expected value of the ith block condition after accounting the relations in the time moment V
σ_i – root mean square of the ith block condition
$P_i(V)$ – ith block failure probability in a time moment V, $P_i(V) = P_i(0) = 0.004$ (according to standards, or the block's documentation)
t_i – mean time between failures of the ith block (according to standards or the block's documentation)
V_i – a time moment of the ith block's replacing with a spare one (0 if the change didn't occur), $V_i := 0$
a_i – an exponential coefficient of the ith block failure probability
b – the critical threshold for a block's parameter values, $b := 0.15$
c_i – the dangerous threshold for the ith block's parameter values
d_i – the threshold for the ith block's parameter values when the block is replaced by a spare one
$\mu_i(V)$ – the membership function of the ith block in the time moment V
$w_i(V)$ – the block condition coefficient in the time moment V
$u_i(V)$ – the coefficient of dangerous/critical blocks proximity to the ith block in the time moment V
k_i – reserving coefficient of the ith block ($k_i = 0$ for no reserve block, $k_i = 1$ for 1 reserve block, $k_i = 2$ for 2 reserve blocks etc.)
$P(V)$ – system failure probability in the time moment V

2) *Computing the root mean square value σ_i for each ith block*

I calculate $x_i = P_i(0) / 2$
I calculate the function's $\Phi(x_i)$ value from the table [1].
I calculate $\sigma_i = -b / \Phi(x_i)$

3) *Computing the exponential coefficient a_i for each ith block*

$a_i = -1 / t_i * \ln(P_i(0))$

4) *Computing the failure probability and the excepted value $m_i(V)$ for each ith block in the time moment V*

I calculate $P_i(V) = \exp(-a_i * (V - V_i))$
I calculate the value of the function $\Phi(P_i(V))$ from the table [1].
I calculate $m_i(\mathrm{V}) = b - \Phi(P_i(V)) * \sigma_i$

5) *Computing the membership function for each ith block*

If $m_i(V) \le -3 * \sigma_i$, then $\mu_i(V) = 1$
If $m_i(V) > -3 * \sigma_i$ and $m_i(V) \le -\sigma_i$, then $\mu_i(V) = \max(\mu_i([-3 * \sigma_i, -\sigma_i]), \mu_i([-2 * \sigma_i, -\sigma_i]))$
If $m_i(V) > -\sigma_i$ and $m_i(V) \le 0$, then $\mu_i(V) = \max(\mu_i([-\sigma_i, 0]), \mu_i([-\sigma_i, 0]))$
If $m_i(V) > 0$ and $m_i(V) \le \sigma_i$, then $\mu_i(V) = \max(\mu_i([0, \sigma_i]), \mu_i([0, \sigma_i]))$
If $m_i(V) > \sigma_i$ and $m_i(V) \le -3 * \sigma_i$, then $\mu_i(V) = \max(\mu_i([\sigma_i, 3 * \sigma_i]), \mu_i([\sigma_i, 2 * \sigma_i]))$
If $m_i(V) \ge 3 * \sigma_i$, then $\mu_i(V) = 1$
If $m_i(V) < -b$ or $m_i(V) > b$, then $w_i(V) = 0$ and $P_i(V) = 1$ (ith block is critical)
If $(m_i(V) > -b$ and $m_i(V) < -c_i)$ or $(m_i(V) > c_i$ and $m_i(V) < b)$, then $w_i(V) = 2$ (ith block is dangerous)
If $m_i(V) \ge -c_i$ or $m_i(V) \le c_i$ then $w_i(V) = 1$ (the block is functional)

6) *Computing the proximity coefficient $u_i(V)$ for each dangerous/critical ith block in the time moment V*

If $w_i(V) = 0$ (ith block is critical), then $u_i(V) = 0$
Furthermore:
If a dangerous and/or critical block is related to the ith, then $u_i(V) = 3$
If a dangerous and/or critical block is related to the ith through a single block, then $u_i(V) = 2$
In all other cases $u_i(V) = 0$
If $w_i(V) = 2$ (ith block is dangerous), then $u_i(V) = 3$
Furthermore:
If a dangerous and/or critical block is related to the ith, then $u_i(V) = 2$
If a dangerous and/or critical block is related to the ith through a single block, then $u_i(V) = 1$

In all other cases $u_i(V) = 0$
I define the related blocks from the matrix **C**.

7) *Computing the addition to the expected value*

$M_i(V) = \sigma_i * w_i(V) * u_i(V) * m_i(V) * \mu_i(V)$

8) *Computing the expected value $m_i(V)$ after accounting the relations in the time moment V*

If $m_i(V) < -d_i$ or $m_i(V) > d_i$, then $m_i(V) = m_i(0)$, else $m_i(V) = m_i(V) + M_i(V)$

9) *Computing the failure probability $P_i(V)$ for each ith block*

If $w_i(V) = 0$ and $k_i = 0$, then $P_i(V) = 1$
If $w_i(V) = 0$ and $k_i = 1$, then $P_i(V) = p_i(V) := 0.004$ and $V_i = V$
If $w_i(V) \neq 0$ and $k_i = 1$ and $m_i(V) < -d_i$ or $m_i(V) > d_i$, then
$V_i = V$, $P_i(V) = p_i(V) := 0.004$, else:
$P_i(V) = \Phi((-b - m_i(V)) / \sigma_i) - \Phi((b - m_i(V)) / \sigma_i)$
I calculate the value of the function Φ according to the table [1].

10) *Computing the system's failure probability*

$$P(V) = (-1)^0 \sum_i (p_i(T)) + (-1)^1 \sum_{ij} (p_i(T) p_j(T)) +$$
$$+ (-1)^2 \sum_{ijk} (p_i(T) p_j(T) p_k(T)) + \ldots + \prod_i (p_i(T))$$

From these values, I draw the graph $P(V)$

11) *If* $P(V) = 1$, *stop, else:*
12) *If* $V \geq T$, *stop, else* $V = V + \Delta$ *and go to step* 4

V. Example

Let us examine a following example system, displayed in the Fig. 2. That is a schematic representation of a universal module for a smart electromechanical system (UN SEMS).
For it, let us assume that, $P_{c0} = 0.996$, $m_{i0} = 0$, $b_i = 0.15$, $t_{0i} = 27000$ hours, $c_i = \sigma_i$, $\Delta = 10000$ hours.

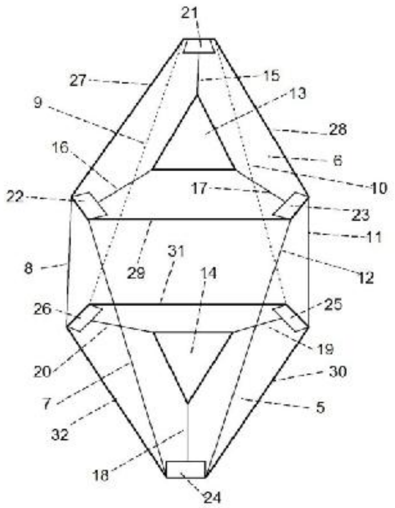

Figure 2. UN SEMS

When displayed as a directed graph on a scheme, the system looks as shown in the Fig. 3.

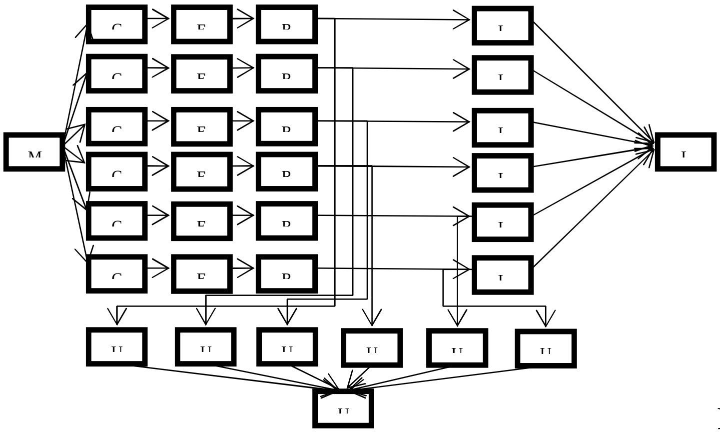

Figure 3. UN SEMS scheme

Here:
MC is Main Controller
LP is Lower Platform
UP is Upper Platform
C1 to C6 are Controllers
E1 to E6 are Engines
R1 to R6 are Reducers
LJ1 to LJ6 are Lower Joints
UJ1 to UJ6 are Upper Joints

In the first example case, let us assume that I don't have spare blocks for any of the system's blocks. Then, after running series of test simulations, I gain the following system probability from step # dependency, which is shown in Fig. 4.

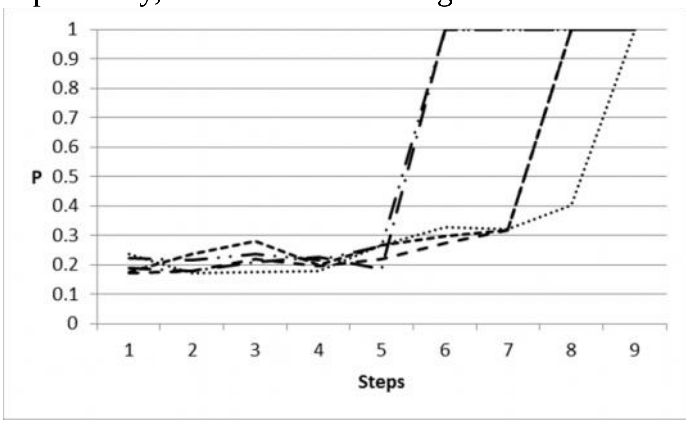

Figure 4. System failure probability graph, without spare blocks

When running series of test simulations assuming that I do have a single spare block for each of the system's blocks, I gain the following dependency, shown in Fig. 5.

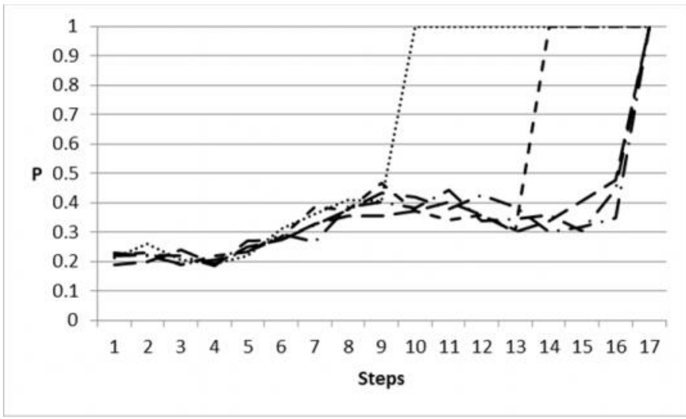

Figure 5. System failure probability graph, with spare blocks

VI. Decision making methods for durability control

Let us examine the pros and cons of the possible decision making methods while controlling the durability of a system with hot reserving.

Let us control the block parameters x_i and switch to a reserve block when $x_i \geq b_i$

Pros: simplicity.

Cons: system hiatus while switching the block and the high probability of false alerts on random short-timed parameter value peaks.

Let us control the block parameters x_i while computing the expected values m_i, and switch to a reserve block when $m_i \geq b_i$

Pros: lower false alert probability.

Cons: system hiatus while switching the block, and the system complexity is higher.

Let us control the block parameters x_i while computing the current expected value m_i, modelling the expected value $m_i(t)$ with the passing of time t **with or without** accounting the relations between the system's blocks, defining by the modelling results the probable time moment T_a of a situation when $m_i \geq b_i$, and in a time moment $T_p = k_p T_a$ ($k_p < 1$) I switch the block to a reserve one and perform the fixing of partial failures.

Pros: low probability of false alerts, low probability of system hiatus while switching the block.

Cons: low prognosis precision of the time moment T_p, high system complexity.

VII. Summary

The proposed modelling method allows to increase the probability of the prognosis of the critical situation occurring time for each block of the system, thus increasing the system durability via timely activation of reserving mechanism. By doing so, I may receive a time reserve to perform the needed technical measures for reserve block switching and for the partial failures fixing.

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